(Gapless chiral) spin liquids in frustrated magnets

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SB, C. Lhuillier, and L. Messio,
Phys. Rev. B 93, 094437 (2016);

SB, L. Messio, B. Bernu, and C. Lhuillier,
Phys. Rev. B 92, 060407(R) (2015).

R. G. Pereira and **SB**, in preparation.

{L. Messio, **SB**, C. Lhuillier, and B. Bernu,
arXiv:1701.01253 (2017).

B. Fåk, **SB**, *et al.*, Phys. Rev. B 95, 060402(R) (2017). }

Reviews: Balents, Nature 464, 199 (2010); Norman, RMP 88, 041002 (2016); Zhou et al., RMP 89, 025003 (2017)

Collaborations

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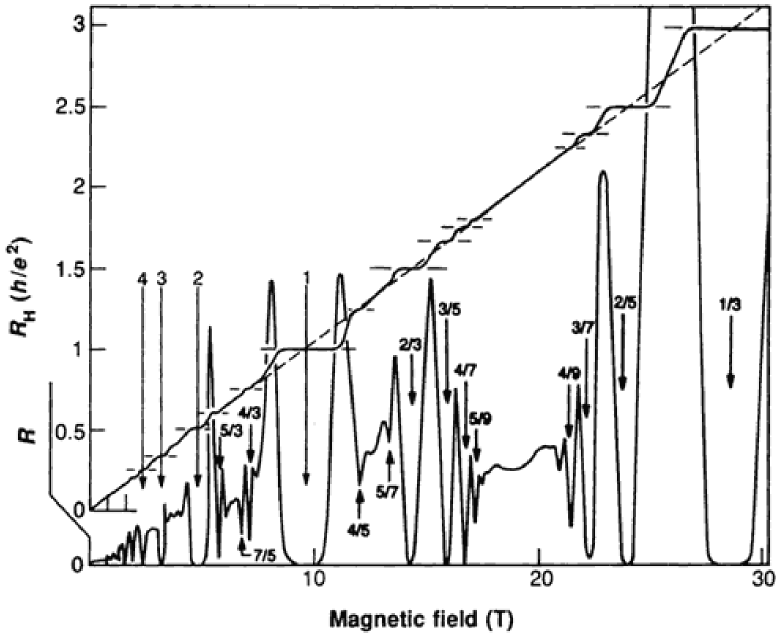
Outline

- Generalities on quantum (spin) liquids; RVB states
- Parton approach; projective symmetry group classification
- Kagome Heisenberg system: kapellasite
- Crossed-chains construction of a gapless chiral spin liquid
- Conclusion

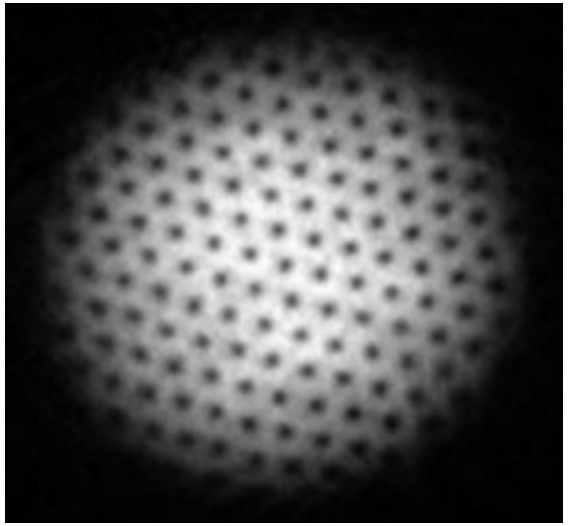
Quantum liquids

- Prominent examples:

Quantum Hall effect



Superfluidity



Here: Liquids beyond Landau / topological order

- No breaking of (continuous, global) symmetry as T -> 0
- Absence of local order parameter

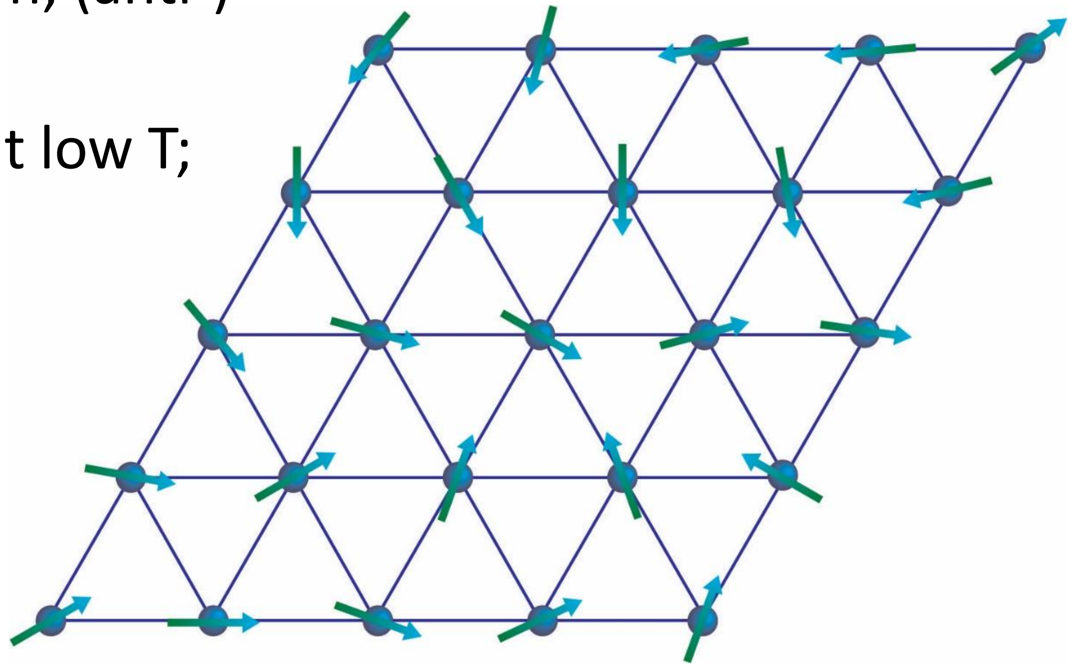
Spin systems (Mott insulators)

Phases:

- **Spin gas**: Independent spins point in random directions; high-T paramagnetic phase.
- **Spin solid**: Freezing of spins to a regular pattern; (anti-) ferromagnetic phase.
- **Spin liquid?** Interacting and fluctuating spins at low T; no ordering and no symmetry breaking

"Cooperative paramagnet"

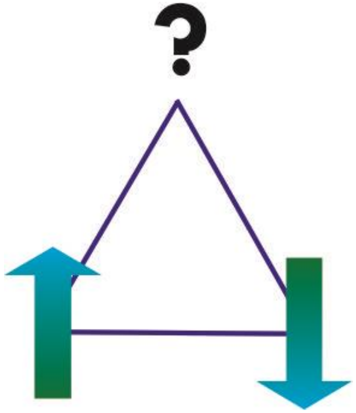
Modern characterization: Long-range entanglement
[Kitaev, Preskill; Levin, Wen 2006]



Geometric frustration

$$H = JS_i^z S_j^z \, , \; J > 0$$

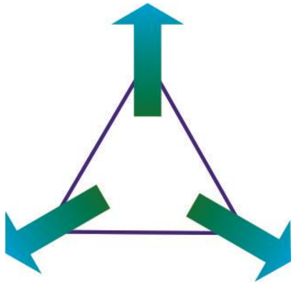
- Two Ising spins:  = Happy
- Three Ising spins with antiferromagnetic interaction:



→Degeneracy of classical ground state.

Triangular Ising lattice [Wannier 1950]

- Classical Heisenberg spins:



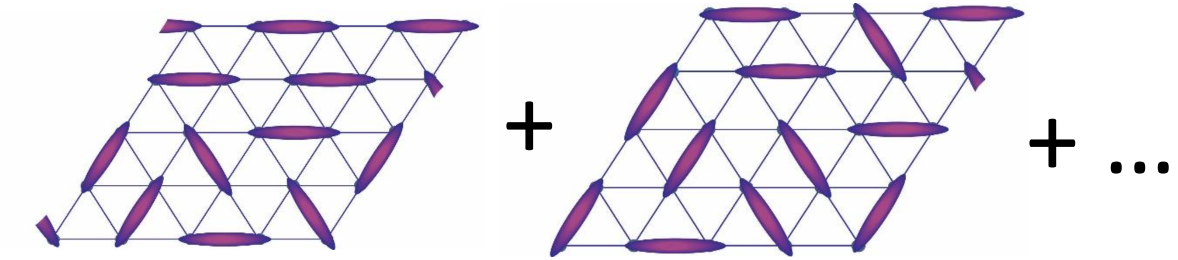
→ Quantum spins?
→ More involved interactions?

Resonating valence bonds (RVB)

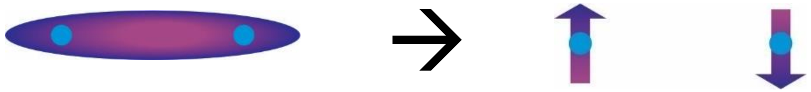
- Valence bond singlet: $|\mathrm{VB}\rangle = \frac{1}{\sqrt{2}}[|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle]$ =  $\langle \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \rangle = -3/4$

  Néel:  $\langle \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \rangle = -1/4$

- Anderson, Fazekas 1973/74: Quantum superposition of valence bonds may beat Néel order

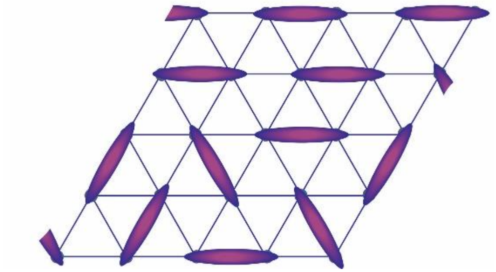


- Anderson, Baskaran 1987/88: **High-temperature superconductivity** can naturally emerge from RVB states (under doping) [Lee, Nagaosa, Wen, RMP 78, 17 (2006)]
- Spinon excitation (spin-1/2); broken valence bond (ΔE = J/2)



Types of valence bond states

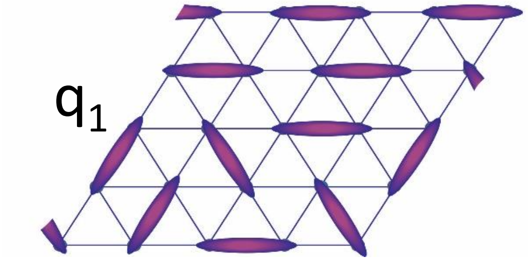
- Valence bond solid



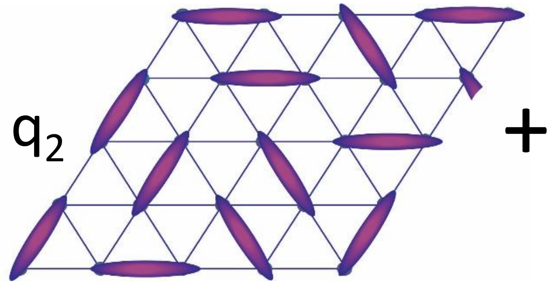
→ Lattice symmetry breaking
→ Product state of valence bonds
→ No long-range entanglement

- Liquid of “short” valence bonds

q_1



$+ \, q_2$

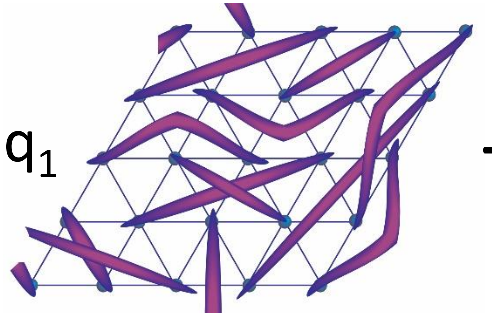


$+ \ldots$

→ No lattice symmetry breaking
→ Long-range entangled
→ Gapped (S=1/2) spinon excitation
→ Spinless vortex excitation (visons)
→ Topological order; group cohomology classification [Chen, Gu, Wen 12]

- Liquid of “long” valence bonds

q_1



$+ \ldots$

→ No lattice symmetry breaking
→ Long-range entangled
→ Low-energy spinon excitations
→ Algebraic/critical correlations (ASL)
→ Refined classification more subtle

Gutzwiller/parton construction of RVB states

- *A priori* it is difficult to make the RVB picture quantitative
- Take simple long-range entangled state – the Fermi gas: $|FS\rangle = \prod_{\epsilon_k<\mu} c^\dagger_{k\downarrow} c^\dagger_{k\uparrow}|0\rangle$

$$P_G\,|FS\rangle = q_1|\uparrow,\downarrow,\downarrow,\uparrow,\ldots\rangle + q_2|\downarrow,0,\uparrow\downarrow,\ldots\rangle + q_3|\downarrow,\uparrow,\uparrow\downarrow,\downarrow,\ldots\rangle + \ldots$$

- Projection (P_G) can efficiently be done (for Fermions) using Monte Carlo tec.
- Liquid character not destroyed by projection ?

[Zhang, Grover, Vishwanath, PRL 2011; Tao Li, EPL 2013]

- Auxiliary degree of freedom [slave particles, partons (spinons)] $c_{j\sigma}$
- Emergent local (gauge) symmetry

→ projective symmetry group

Projective symmetry group

- How to classify RVB spin states beyond symmetry breaking?
  - Broken symmetry: Landau theory; Bragg-peaks, Anderson TOF
- X.-G. Wen: Parton classification [PRB 65, 165113 (2002)]
- Parton classification of ***chiral*** spin liquid states [**SB** et al., PRB 93, 094437 (2016)]

$$\chi = \langle \boldsymbol{S}_1 \cdot (\boldsymbol{S}_2 \times \boldsymbol{S}_3) \rangle \neq 0 \qquad \Rightarrow \Theta \text{ and } \sigma(2 \leftrightarrow 3) \text{ broken}$$

SR invariant: $\langle \boldsymbol{S} \rangle = 0$

Kalmeyer and Laughlin, PRL 59, 2095 (1987).

Wen, Wilczek, Zee, PRB 39, 11413 (1989).

Yang, Warman, Girvin, PRL 70, 2641 (1993).

Parton construction & classification

Spin-1/2 Heisenberg model: $H = \sum_{ij} J_{ij}\, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$

(a) Fractionalize spin into spinons f_α, carrying ΔS = 1/2 (magnons ΔS=1)
(f_α: "Abrikosov fermion" creation operator [JETP 26, 641 (1968)])

spinon doublet: $\boldsymbol{f} = (f_\uparrow, f_\downarrow)^T$ $\quad$ $2S_a = \boldsymbol{f}^\dagger \sigma_a \boldsymbol{f}$ $\quad$ $\boldsymbol{S}^2 = \frac{3}{4} n[2-n]$

enlarged local Hilbert space:

$\{|\uparrow\rangle, |\downarrow\rangle\} \Rightarrow \{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle\}$ $\quad$ constraint/physical subspace: $n = \boldsymbol{f}^\dagger \boldsymbol{f} \equiv 1$

gauge doublet: $\boldsymbol{\psi} = (f_\uparrow, f_\downarrow^\dagger)^T$

gauge transformation: $\boldsymbol{\psi} \mapsto g\boldsymbol{\psi},\ g \in \mathrm{SU}(2)$: leaves spin S_a invariant

[Affleck et al, PRB 38, 745 (1988)]
[Marston et al, PRB 39, 11538 (1989)]

Emergent SU(2) symmetry is local:
(gauge sym; ≠ spin rot!)

$$\boldsymbol{\psi} = (f_\uparrow, f_\downarrow^\dagger)^T$$

$$\boldsymbol{\psi} \mapsto g\boldsymbol{\psi},\ g \in \mathrm{SU}(2)$$

Projective symmetry group:
How can actual symmetries be represented in the spinon Hilbert space?

[X.-G. Wen, PRB 65, 165113 (2002)]

e.g., time-reversal: $\Theta(\boldsymbol{\psi}) = \varepsilon\boldsymbol{\psi}^* \xrightarrow{g_\Theta = \varepsilon^T} \boldsymbol{\psi}^*$ $\qquad \varepsilon = i\sigma_2$

Algebraic relations among symmetries must be *respected* by the representation (up to gauge transformations) !

Parton Ansatz

$$H = \sum_{ij} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \qquad 2S_a = \boldsymbol{f}^\dagger \sigma_a \boldsymbol{f}$$

+ Hubbard-Stratonovich
or MF decoupling

⇨ (b) Quadratic spinon Hamiltonian (= singlet "ansatz")

$$H_0 = \sum_{ij} \xi_{ij} f^\dagger_{i\alpha} f_{j\alpha} + \Delta_{ij} f_{i\uparrow} f_{j\downarrow} + \text{h.c.} = \sum_{ij} \boldsymbol{\psi}^\dagger_i u_{ij} \boldsymbol{\psi}_j + \text{h.c.} \qquad \boldsymbol{\psi} = (f_\uparrow, f^\dagger_\downarrow)^T$$

Ansatz: $u = \{u_{ij}\}$

$$u_{ij} = \begin{pmatrix} \xi_{ij} & \Delta_{ij} \\ \Delta^*_{ij} & -\xi^*_{ij} \end{pmatrix}$$

Interpretations/uses of $H_0(u)$:

(i) Low-energy effective theory;
Invariant gauge group (IGG_u): U(1) [$f_j \mapsto e^{i\varphi} f_j$] or $\mathbb{Z}_2$ [$f_j \mapsto -f_j$]

(ii) Self-consistent saddle point solutions for H

(iii) Tool for constructing spin w.f. by Gutzwiller projection : $|\psi\rangle = \prod_j n_j[2-n_j] |\psi_0\{u_{ij}\}\rangle$

Variational Monte Carlo (VMC) method

Projective symmetry group (PSG)

1. Algebraic PSG: Representation classes of the symmetry group SG in the gauge group $\mathcal{G}=\{g\},\ g=\otimes g_j\ ,\ g_j \in \mathrm{SU}(2)$

$$Q\colon \mathrm{SG} \to \mathcal{G}$$
$$x \mapsto g_x$$

Equivalence of reps:

$$Q^1 \sim Q^2 \iff \exists g \in \mathcal{G}\ \text{ s.t. }\ Q^1 = gQ^2 g^\dagger$$

Algebraic relations in SG respected *up to the IGG*, e.g.: reflection $\sigma^2 = 1$ ⟹ $g_\sigma(\boldsymbol{r})g_\sigma(\sigma\boldsymbol{r}) \in \mathrm{IGG}\ \{\pm 1\}$

IGG: Invariant Gauge Group (subgrp of $\mathcal{G}$)
(here: $\mathbb{Z}_2$ classification)

2. Invariant PSG: Ansatz u respecting SG for each PSG class

action of symmetry x on Ansatz: $Q_x(u_{ij}) = (-)^{\tau_x} g_x(i) u_{x^{-1}(ij)} [g_x(j)]^\dagger$

$$Q_x(u) = u \qquad \text{for all } x \text{ in SG}$$

PSG: Kagome lattice

SB et al., Phys. Rev. B 92, 060407(R) (2015)

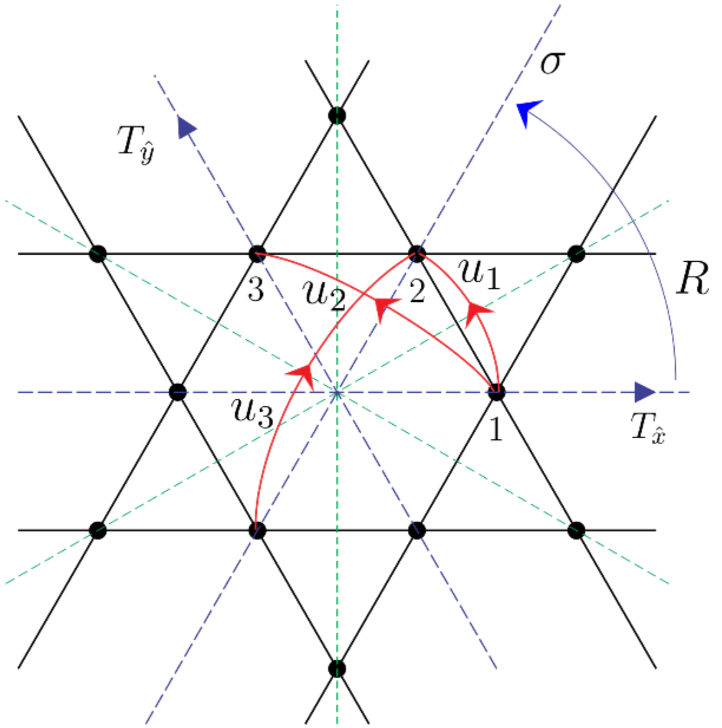
SB, C. Lhuillier, and L. Messio, Phys. Rev. B 93, 094437 (2016).

Symmetries: $\mathrm{SG}_{\tau_\sigma,\tau_R} = \{T_{\hat{x}}, T_{\hat{y}}, \sigma\Theta^{\tau_\sigma}, R\Theta^{\tau_R}\}$

$\tau_R = 0, \tau_\sigma = 0$: Symmetric QSL

$\tau_R = 0, \tau_\sigma = 1$: "Kalmeyer-Laughlin" CSL

$\tau_R = 1$: Staggered-flux CSL



$g_x = \mathbb{1}_2$

$g_y = (\epsilon_2)^x \mathbb{1}_2$

$g_\sigma(x,y) = (\epsilon_2)^{xy} g_\sigma$

$g_R(x,y) = (\epsilon_2)^{xy+y(y+1)/2} g_R$

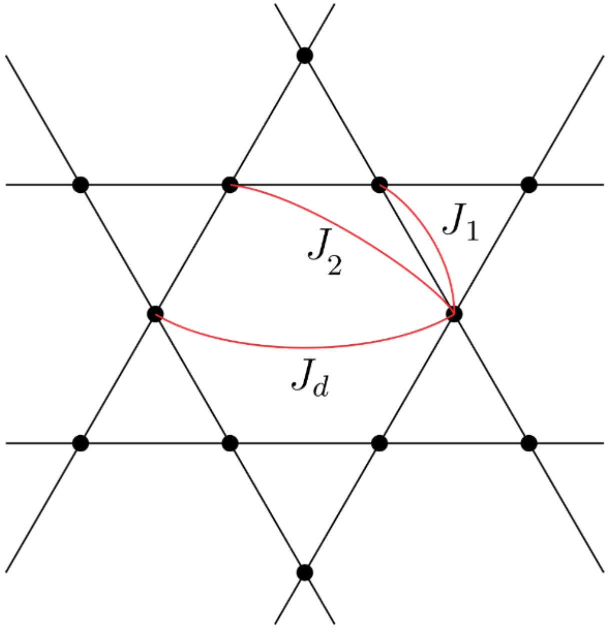
$\epsilon_2 = \pm 1$

no.	g_σ	g_R	ϵ_σ	$\epsilon_{R\sigma}$	ϵ_R	sym
1	$\mathbb{1}_2$	$\mathbb{1}_2$	+	+	+	SU(2)
2	$i\sigma_3$	$\mathbb{1}_2$	−	−	+	U(1)
3	$\mathbb{1}_2$	$i\sigma_3$	+	−	−	U(1)
4	$i\sigma_3$	$i\sigma_3$	−	+	−	U(1)
5	$i\sigma_2$	$i\sigma_3$	−	−	−	$\mathbb{Z}_2$

⇨ **10 PSG classes** on kagome

Material: kapellasite [$ZnCu_3(OH)_6Cl_2$]

- No ordering down to mK, gapless continuum of spin excitations
- Weak ferro Curie-Weiss temp Θ_{CW} ~ 9 K
- Farther-neighbor Heisenberg exchange: J_1 ~ -12 K, J_2 ~ -4 K, J_d ~ 16 K
- Powder samples



R. H. Colman et al, C.M. 20, 6897 (2008); 22, 5774 (2010).

O. Janson at al, PRL 101, 106403 (2008).

H. O. Jeschke et al, PRB 88, 075106 (2013).

E. Kermarrec et al, PRB 90, 205103 (2014).

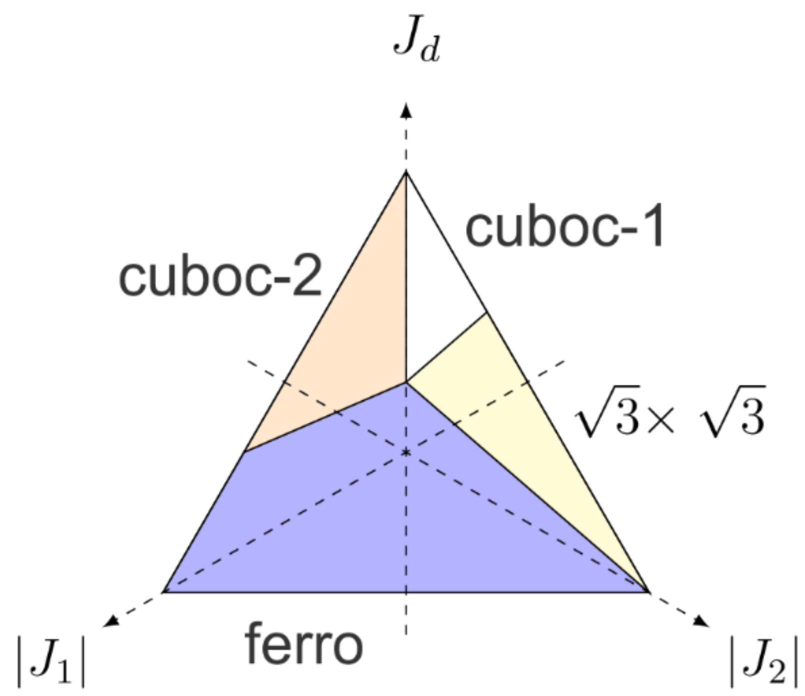
B. Fåk et al, PRL 109, 037208 (2012).

B. Bernu et al, PRB 87, 155107 (2013).

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_d \sum_{\langle\langle i,j \rangle\rangle_d} \mathbf{S}_i \cdot \mathbf{S}_j$$

→ Experimental evidence for a gapless quantum spin liquid

Classical J_1-J_2-J_d kagome Heisenberg model



$$|J_1|+|J_2|+J_d = 1$$

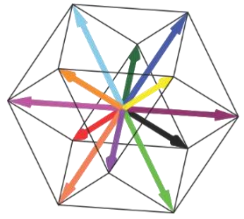
$$J_1 < 0,\ J_2 < 0,\ J_d > 0$$

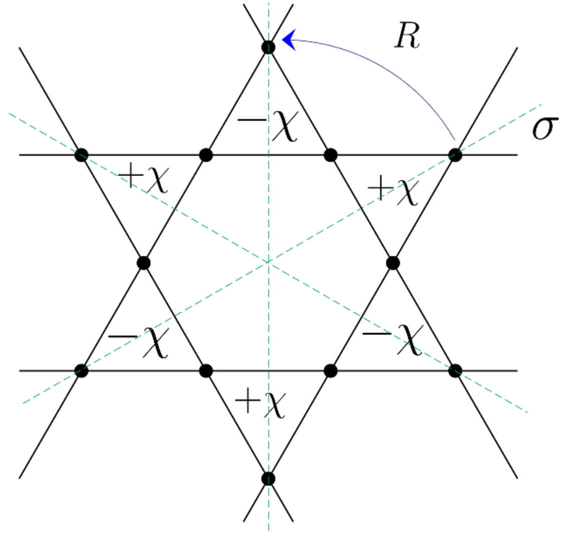
L. Messio *et al.*, PRB 83, 184401 (2011).

cuboc-1,-2: non-planar Néel order with $\chi = \boldsymbol{S}_1\cdot(\boldsymbol{S}_2\times\boldsymbol{S}_3) \neq 0$

12 site unit cell

Spontaneous breaking of time-reversal, (up to) lattice reflection and rotation





What happens in the case of quantum spin S=1/2?

Is the elusive chiral spin liquid realized in kapellasite?

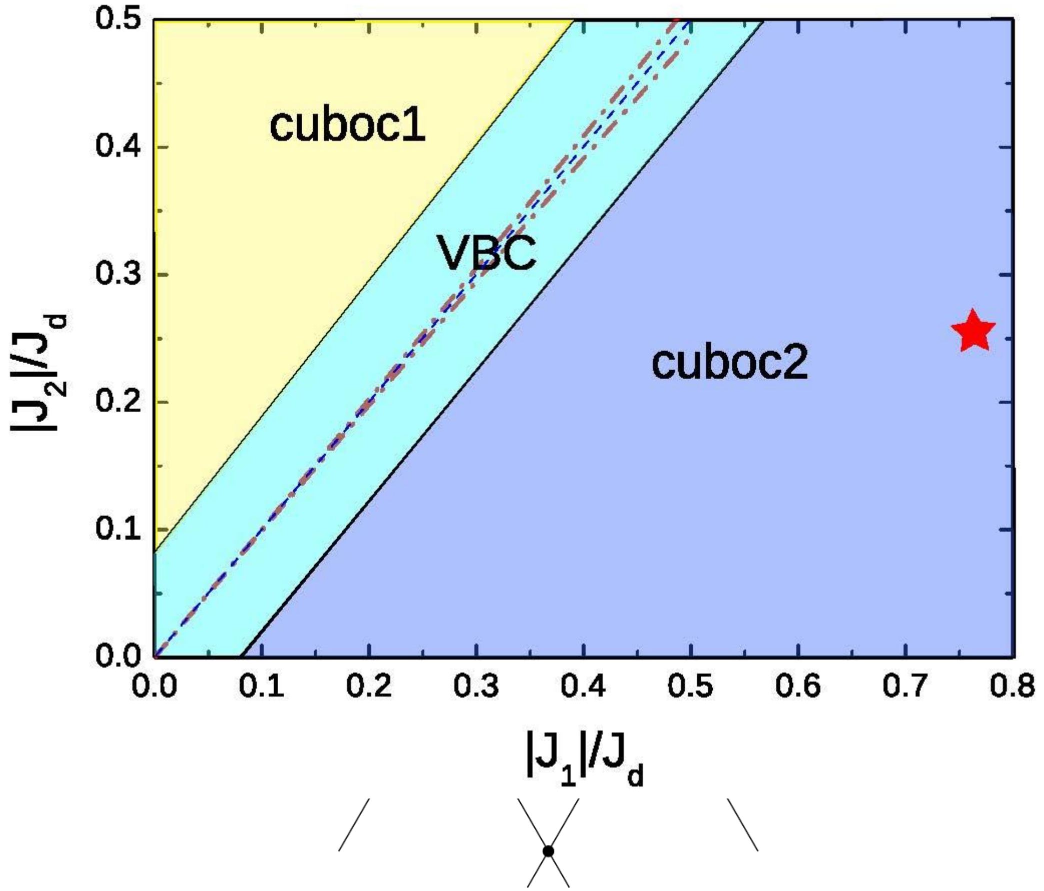
Kalmeyer and Laughlin, PRL 59, 2095 (1987).

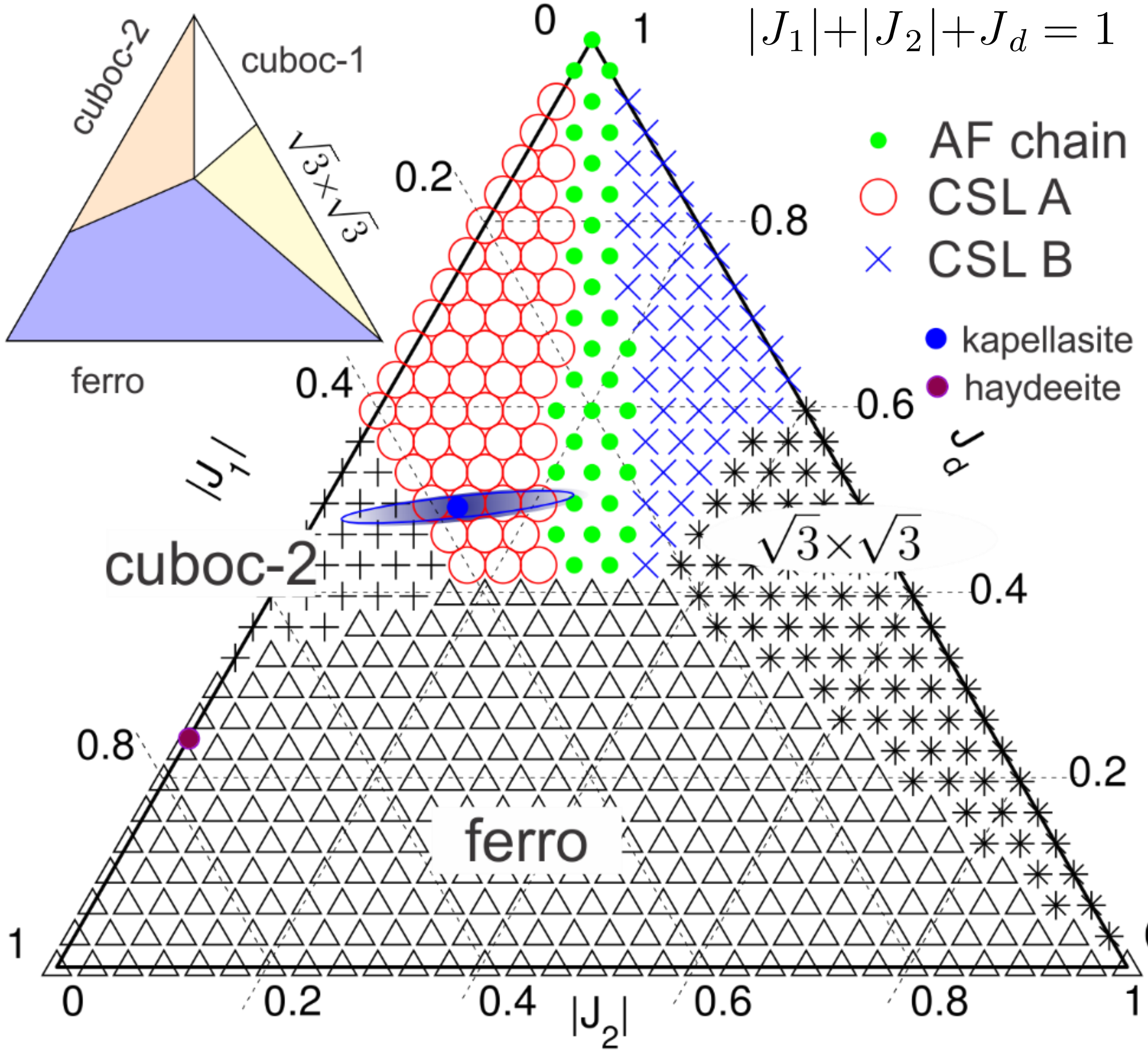
Wen, Wilczek, Zee, PRB 39, 11413 (1989).

Yang, Warman, Girvin, PRL 70, 2641 (1993).

Quantum phase diagram - VMC

SB et al., PRB 92, 060407 (2015)
DMRG by Gong et al, PRB 2016





Spin model:

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_d \sum_{\langle i,j \rangle_d} \mathbf{S}_i \cdot \mathbf{S}_j$$

with

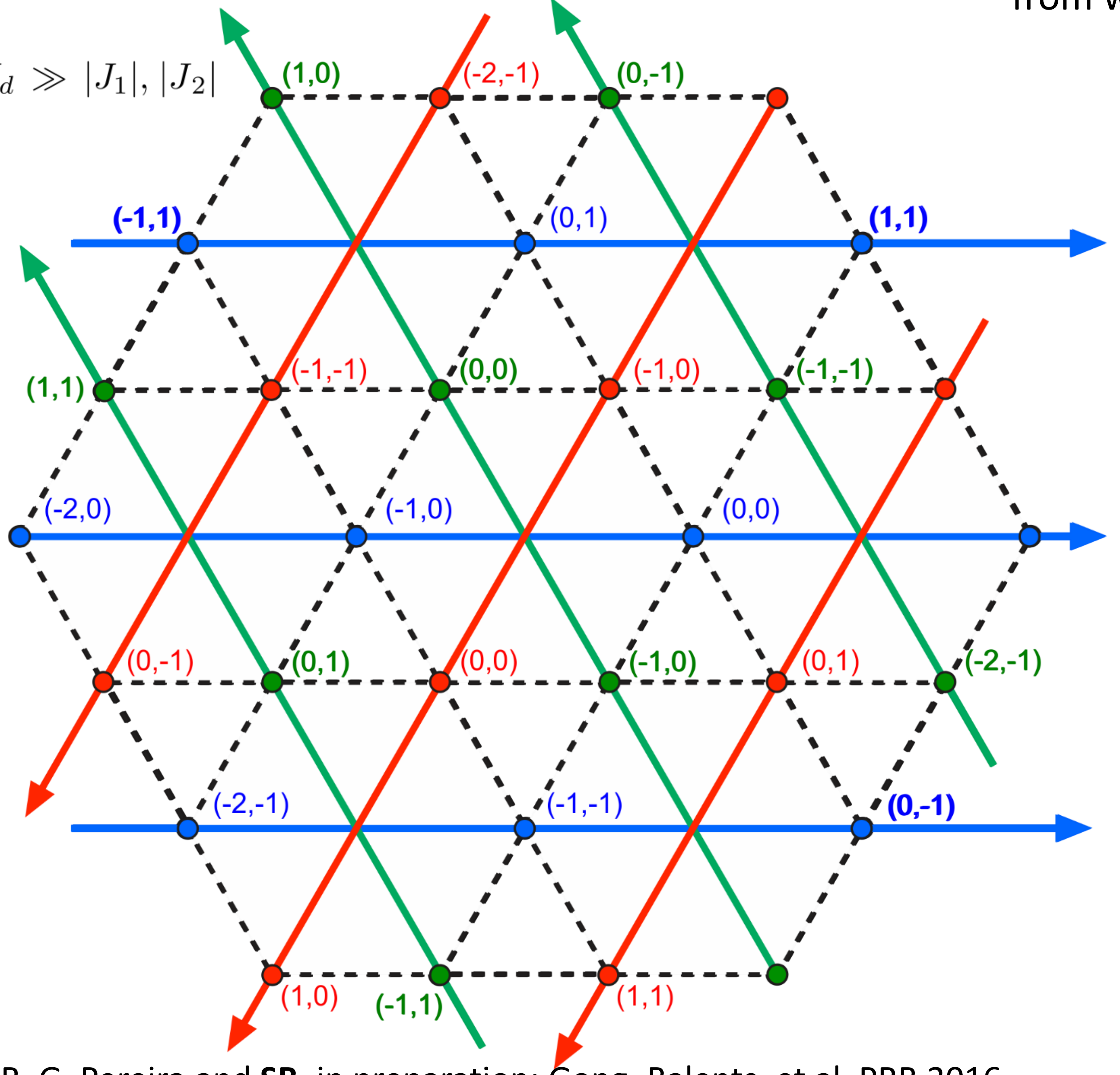
$$J_1 < 0,\ J_2 < 0,\ J_d > 0$$

Iqbal, Valenti, Greiter, Thomale, et al, PRB 2015
Gong, Zhu, Yang, Starykh, Sheng, Balents, PRB 2016:
DMRG → ordered phases

Crossed-chains model

Kalmayer-Laughlin CSLs from wire construction: Neupert, Chamon, Mudry, Thomale, PRB (2014)
Meng et al, PRB 241106 (2015)
Lecheminant, Tsvelik, arXiv:1608.05977

$J_d \gg |J_1|, |J_2|$



$$\mathbf{S}_q(j,l) \sim a_{\|}[\mathbf{J}_{Lq}(x,l) + \mathbf{J}_{Rq}(x,l) + (-1)^j \mathbf{n}_q(x,l)]$$

$$q \in \{\text{red,green,blue}\} = \{1,2,3\}$$

Dimerization op: $(-)^j \mathbf{S}_q(j,l)\cdot\mathbf{S}_q(j+1,l) \sim \varepsilon_q(x,l)$

$$H_0 \sim \sum_{q,l} \frac{2\pi v}{3} \int dx [\mathbf{J}^{\,2}_{qL}(x,l) + \mathbf{J}^{\,2}_{qR}(x,l)] \qquad v = \pi J_d a$$

Transverse direction: even/odd fields

$$n_q(j,l) \sim n_q^e(j,y) + (-)^l n_q^o(j,y)$$

$$H'_n \sim (J_2 - J_1) \int d^2x \; \mathbf{n}_q^o(-x,y)\cdot\mathbf{n}^{\,e}_{q+1}(x+y,x)$$

→ nonplanar magnetic order (cuboc)

$$H'_\varepsilon \sim J_1 J_2 \int d^2x \; \varepsilon_q^o(-x,y)\varepsilon^e_{q+1}(x+y,x)$$

→ valence-bond-crystal

R. G. Pereira and **SB**, in preparation; Gong, Balents, et al, PRB 2016.

Crossed-chains model

R. G. Pereira and **SB**, in preparation

[Sen and Chitra, PRB 1995]
[Bauer, Trebst, Ludwig, et al, Nat. Com. 2014; arXiv:1302.6963]
[Wieteck, Läuchli, PRB 2017]
[Gorohovsky, Pereira, Sela, PRB 2015]

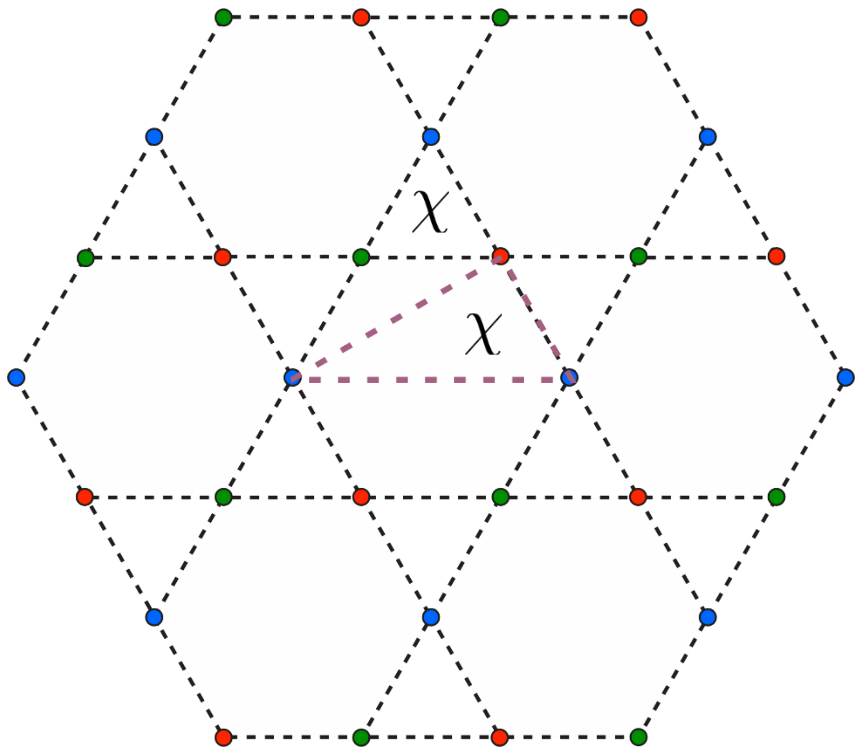
$$J_1 = J_2 \qquad |J_1|, J_\chi \ll J_d$$

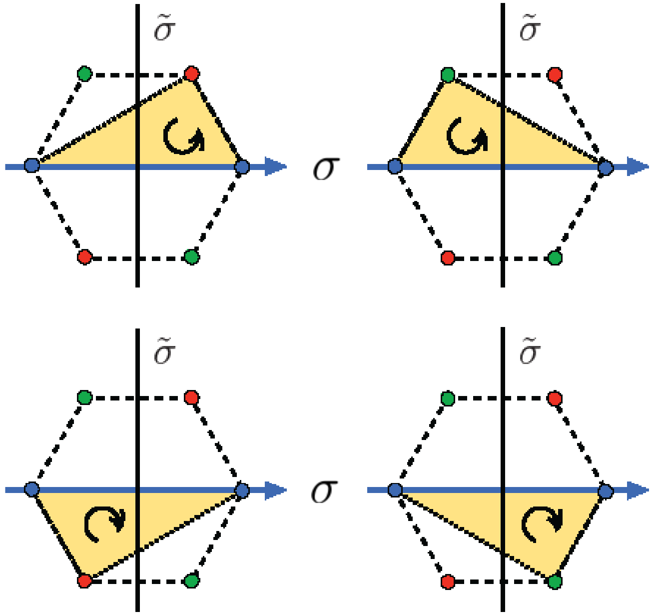
Add chiral three-site interaction:

$$H_\chi = J_\chi \sum_{ijk\in\Delta} \mathbf{S}_i\cdot(\mathbf{S}_j\times\mathbf{S}_k)$$

→Small triangles: Only generates **irrelevant** interchain couplings:

$$\delta H \sim \mathbf{J}_{1R}\cdot(\mathbf{J}_{2R}\times\mathbf{J}_{3R})$$





J_1–J_2–J_d-triangles: two sites belong to the same chain → marginal coupling.

→ Breaks time reversal and reflection $\tilde\sigma$; preserves σ along the chains;
Rotation C_6 broken down to C_3.

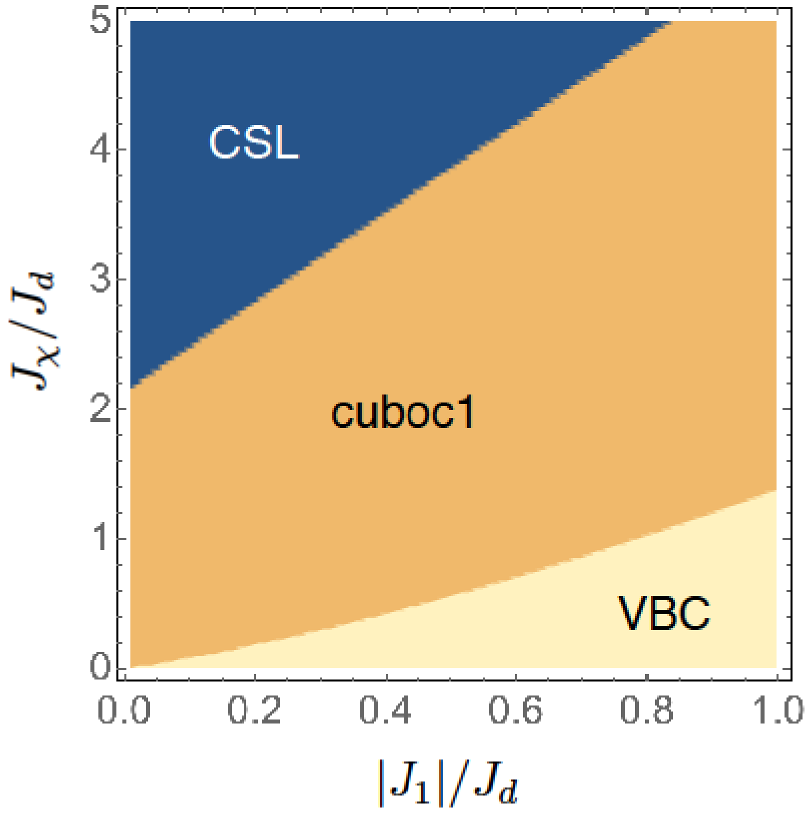
$$\delta H \sim (J_1 \mp J_\chi)\int d^2x\ \mathbf{J}^e_{q,L/R}(-x,y)\cdot\mathbf{J}^e_{q+1,L/R}(x+y,x)$$

Crossed-chains model

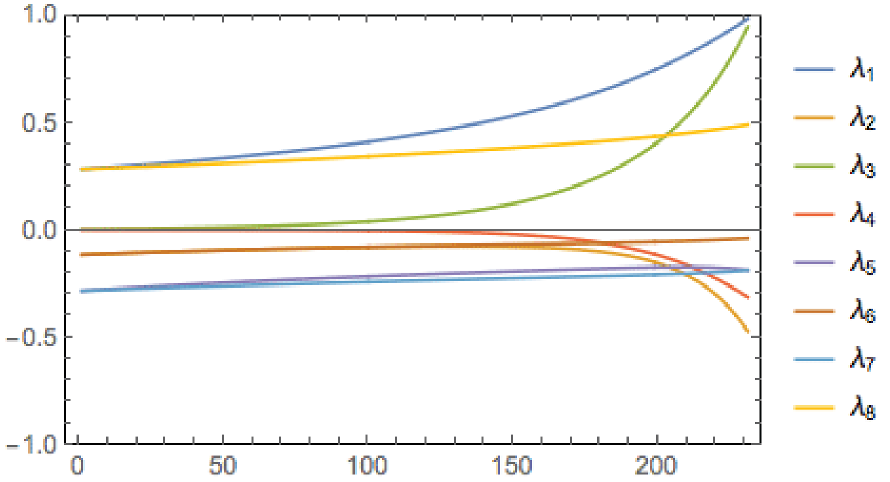
$$\delta H = \sum_j \mathcal{H}_j$$

$$\begin{aligned}
\mathcal{H}_1(x,y) &= 2\pi v \lambda_1 \alpha^{-1}\, \varepsilon^o_q(-x,y)\varepsilon^e_{q+1}(x+y,x) \\
\mathcal{H}_2(x,y) &= 2\pi v \lambda_2 \alpha^{-1}\, \mathbf{n}^o_q(-x,y)\cdot\mathbf{n}^e_{q+1}(x+y,x) \\
\mathcal{H}_3(x,y) &= 2\pi v \lambda_3\, \mathbf{J}^e_{qL}(x,y)\cdot\mathbf{J}^e_{qR}(x,y) \\
\mathcal{H}_4(x,y) &= 2\pi v \lambda_4\, \mathbf{J}^o_{qL}(x,y)\cdot\mathbf{J}^o_{qR}(x,y) \\
\mathcal{H}_5(x,y) &= 2\pi v \lambda_5\, \mathbf{J}^e_{qL}(-x,y)\cdot\mathbf{J}^e_{q+1,L}(x+y,x) \\
\mathcal{H}_6(x,y) &= 2\pi v \lambda_6\, \mathbf{J}^e_{qR}(-x,y)\cdot\mathbf{J}^e_{q+1,R}(x+y,x) \\
\mathcal{H}_7(x,y) &= 2\pi v \lambda_7\, \mathbf{J}^e_{qL}(-x,y)\cdot\mathbf{J}^e_{q+1,R}(x+y,x) \\
\mathcal{H}_8(x,y) &= 2\pi v \lambda_8\, \mathbf{J}^e_{qR}(-x,y)\cdot\mathbf{J}^e_{q+1,L}(x+y,x)
\end{aligned}$$

$$\lambda^0_{5,6} \sim (J_1 \mp J_\chi)$$

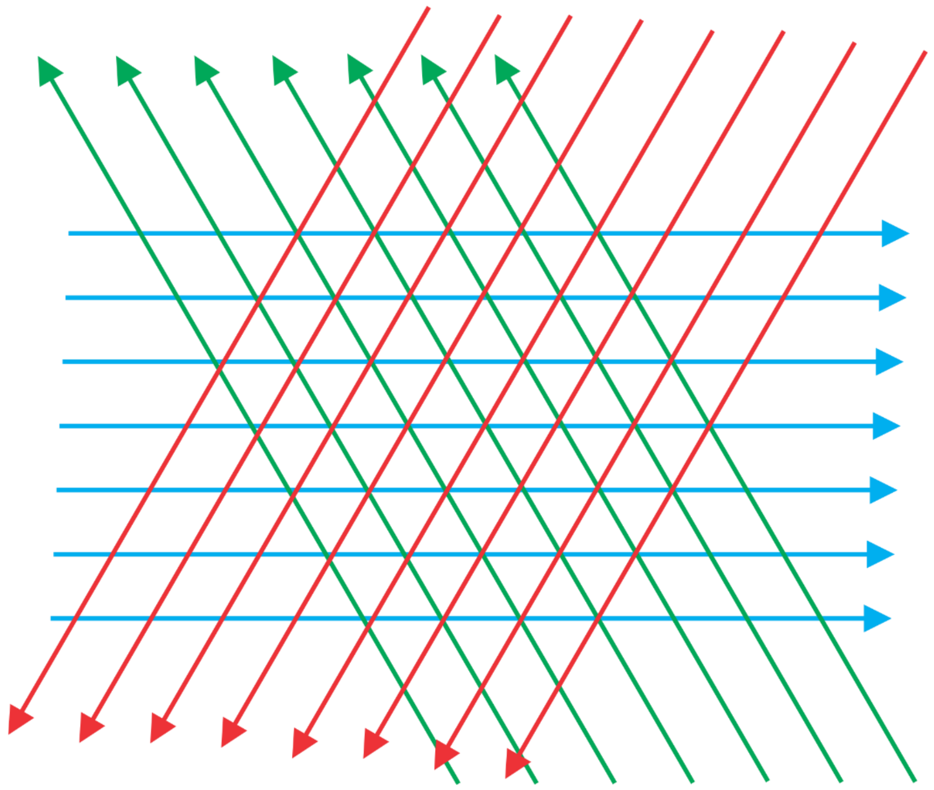


Identify running λ_j that reaches strong coupling **first** in the RG flow:



$$\lambda_6\, \mathbf{J}_{qR}(-x,y)\cdot\mathbf{J}_{q+1,R}(x+y,x) \;\sim\; \cos\{\sqrt{4\pi}[\varphi_{qR}(-x,y)-\varphi_{q+1,R}(x+y,x)]\}$$

→ R boson gap $\sim\; v/\alpha^*$ in all chains, L remain gapless



Physical properties:

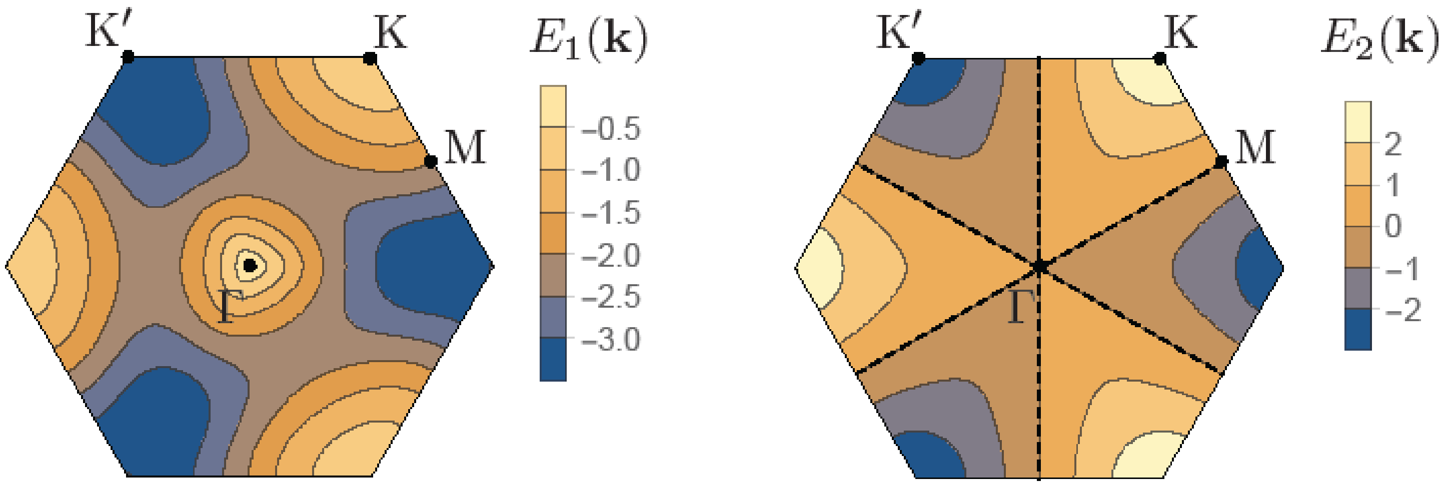
- Gapless chiral bulk spin currents
- Vanishing thermal Hall response
- Linear-T specific heat
- Stable to weak disorder (irrelevant)
- Power-law spin-spin correlations: $\langle \mathbf{S}_q(x,y)\cdot\mathbf{S}_q(x+r,y)\rangle \;\sim\; -r^{-2}$
- Area law entanglement entropy: $S_A(\ell) \;\sim\; c_L\, \ell\, \log(\ell)$

→ Spinon/parton Fermi surface?

Crossed-chains model: parton construction

Complex Fermionic parton theory - PSG : [SB et al, PRB 2016]

$$\tau_\sigma = 0,\ \tau_R = 1;\ \epsilon_2 = 1,\ g_\sigma = g_R = \mathbb{1}_2$$
$$t_1 = i, t_2 = 1, t_d = i$$



Similarly: Majorana fermion fractionalization

[Kitaev, Ann. Phys. 2006]
[Biswas, Fu, Laumann, Sachdev, PRB 2011]

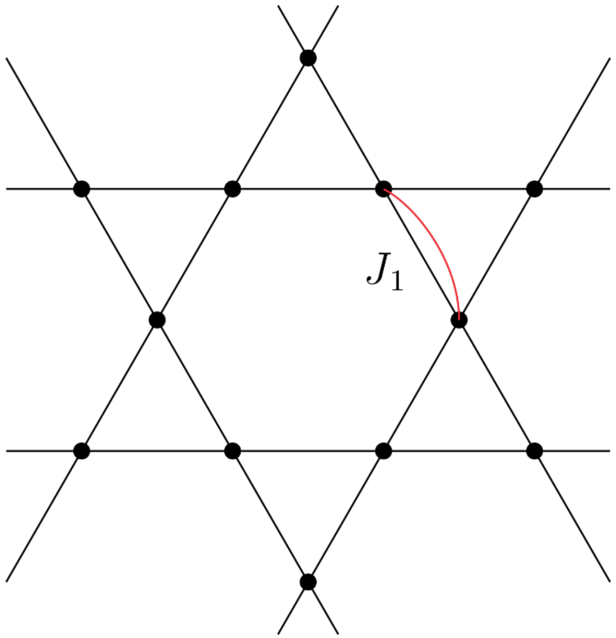
Conclusion & outlook

- RVB states, parton construction, PSG classification
- Exhaustive list of fermonic parton CSLs (kagome, triangular)
- Exotic phases in a S=1/2 kagome system (kapellasite)
- Crossed-chains construction for dominant Jd
- Outlook:
  - Spin-orbit coupled models, DM interation
  - Microscopic properties of parton states
  - PSG og nonsymorphic/higher-D space groups
  - Can we marry fRG with parton PSG?

Thank you!

Herbertsmithite [$ZnCu_3(OH)_6Cl_2$]

- No ordering down to mK, gapless/gapped (?) spin excitations
- Strong AF Curie-Weiss, dominant $J_1 = J \sim 200$ K ; $J_2 = J_d = 0$
- Single crystals [Young Lee (MIT), now also France (Ph. Mendels)]
- Perturbations; e.g. Dzialoshinskii-Moriya: $D_z \sim 0.8$ J



T.-H. Han et al, Nature 492, 406 (2012).

A. Zorko et al, PRL 118, 017202 (2017).

A. Zorko et al, PRL 101, 026405 (2008).

I. Dzyaloshinsky, J. Phys. Chem. Solids 4, 241 (1958).

T. Moriya, PRL 4, 228 (1960).

L. Shekhtman et al, PRL 69, 836 (1992).

Spin-orbit coupling/Dzyaloshinskii-Moriya (DM) Interaction:

$$H = J \sum_{\langle i,j \rangle} h_{ij}$$

$$h_{ij} = \boldsymbol{S}'_i \cdot \boldsymbol{S}'_j$$

DM-vector: $\boldsymbol{D}_{ij} = J\,\theta_{ij}\,\hat{\boldsymbol{d}}_{ij}$

$$\boldsymbol{S}'_i = \boldsymbol{S}_i \text{ rotated by } -\theta_{ij} \text{ around } \hat{\boldsymbol{d}}_{ij}$$

$$\boldsymbol{S}'_j = \boldsymbol{S}_j \text{ rotated by } +\theta_{ij} \text{ around } \hat{\boldsymbol{d}}_{ij}$$

Schwinger-boson mean-field theory (for DM)

L. Messio, **SB**, et al, arXiv:1701.01253

(a) Fractionalize spin into bosonic ΔS = 1/2 spinons b_α

(b_α: "Schwinger boson" creation operator)

$$2S_a = \boldsymbol{b}^\dagger \sigma_a \boldsymbol{b} \qquad \boldsymbol{b} = (b_\uparrow, b_\downarrow)^T \qquad \boldsymbol{S}^2 = \frac{1}{4} n(2+n)$$

enlarged local Hilbert space: $\{|\uparrow\rangle, \ |\downarrow\rangle\} \Rightarrow \{|0\rangle, |1,0\rangle, |0,1\rangle, |1,1\rangle, \ldots\}$

constraint/physical subspace:

$$n = \boldsymbol{b}^\dagger \boldsymbol{b} \equiv 2S$$

emergent gauge symmetry: $\boldsymbol{b} \mapsto g\boldsymbol{b}$, $\quad g \in \mathrm{U}(1)$

Quadratic spinon theories: $h_{ij} = :B_{ij}^\dagger B_{ij}: - A_{ij}^\dagger A_{ij}$

$$A_{ij} = e^{-i\theta_{ij}} b_{i\uparrow} b_{j\downarrow} - e^{i\theta_{ij}} b_{i\downarrow} b_{j\uparrow}$$

$$B_{ij} = e^{i\theta_{ij}} b_{i\uparrow}^\dagger b_{j\uparrow} - e^{-i\theta_{ij}} b_{i\downarrow}^\dagger b_{j\downarrow}$$

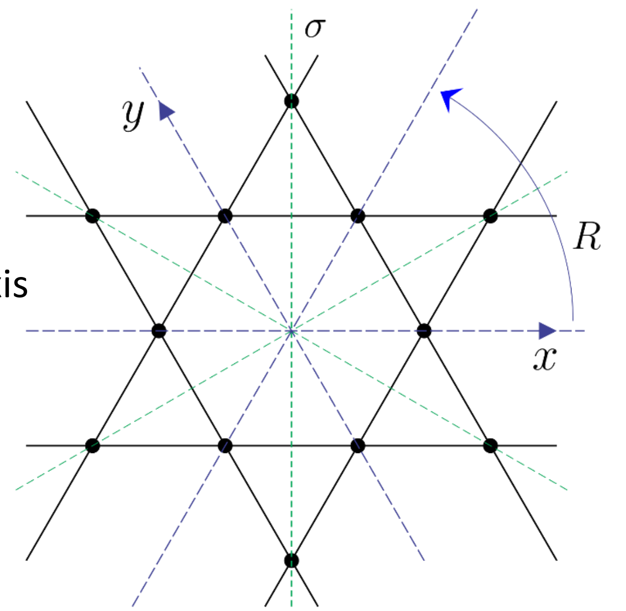
$$\Rightarrow \quad h_{ij}^{\mathrm{MF}} = \mathcal{B}_{ij}^* B_{ij} - \mathcal{A}_{ij}^* A_{ij} + \mathrm{h.c.}$$

$$\mathrm{SG} = \{T_{\hat{x}}, T_{\hat{y}}, R\Theta^{\tau_R}, \sigma S_{\pi x}\Theta^{\tau_\sigma}\}$$

$S_{\pi x}$: spin-rotation π around x-axis

$\tau_R = 1$ or $\tau_\sigma = 1$

⇒ spont. breaking of pt group and TR: CSL



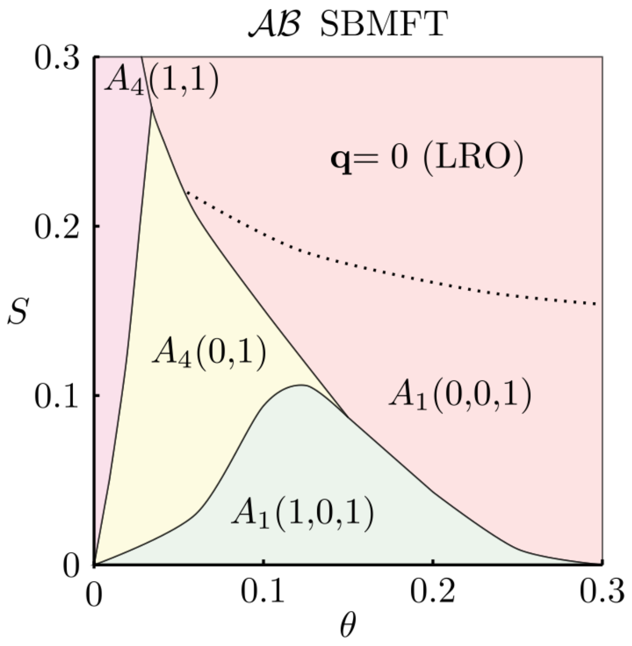
(b) PSG classification of MF states:
L. Messio et al, PRB 83, 184402 (2011).

Self-consistent solutions

$$\mathcal{A}_{ij} = \langle A_{ij} \rangle \qquad A_{ij} = e^{-i\theta_{ij}} b_{i\uparrow} b_{j\downarrow} - e^{i\theta_{ij}} b_{i\downarrow} b_{j\uparrow}$$

$$\mathcal{B}_{ij} = \langle B_{ij} \rangle \qquad B_{ij} = e^{i\theta_{ij}} b^\dagger_{i\uparrow} b_{j\uparrow} - e^{-i\theta_{ij}} b^\dagger_{i\downarrow} b_{j\downarrow}$$

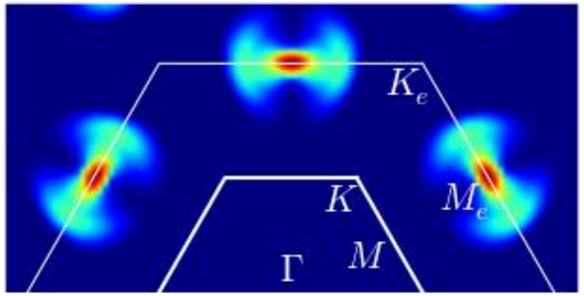
$$2S = \sum_j \langle n_j \rangle$$



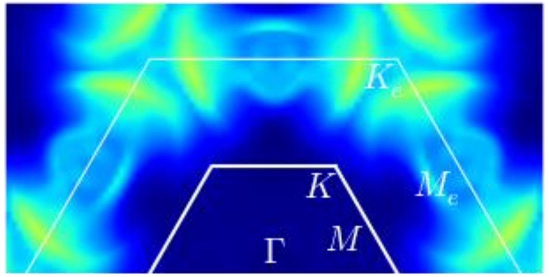
L. Messio, **SB**, C. Lhuillier, B. Bernu, arXiv:1701.01253

$A_4(1,1)$ phase: L. Messio et al, PRL 108, 207204 (2012)

$S(\boldsymbol{q}, \omega)$ in new CSL phase $A_4(0,1)$

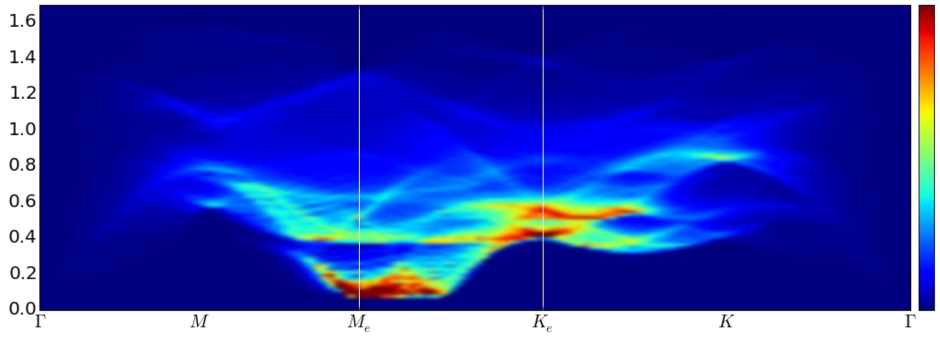


$\omega = 0-0.15J$



$\omega = 0.3-0.45J$

ω/J



$Ba_3NiSb_2O_9$

PRL 107,197204 (2011) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2011

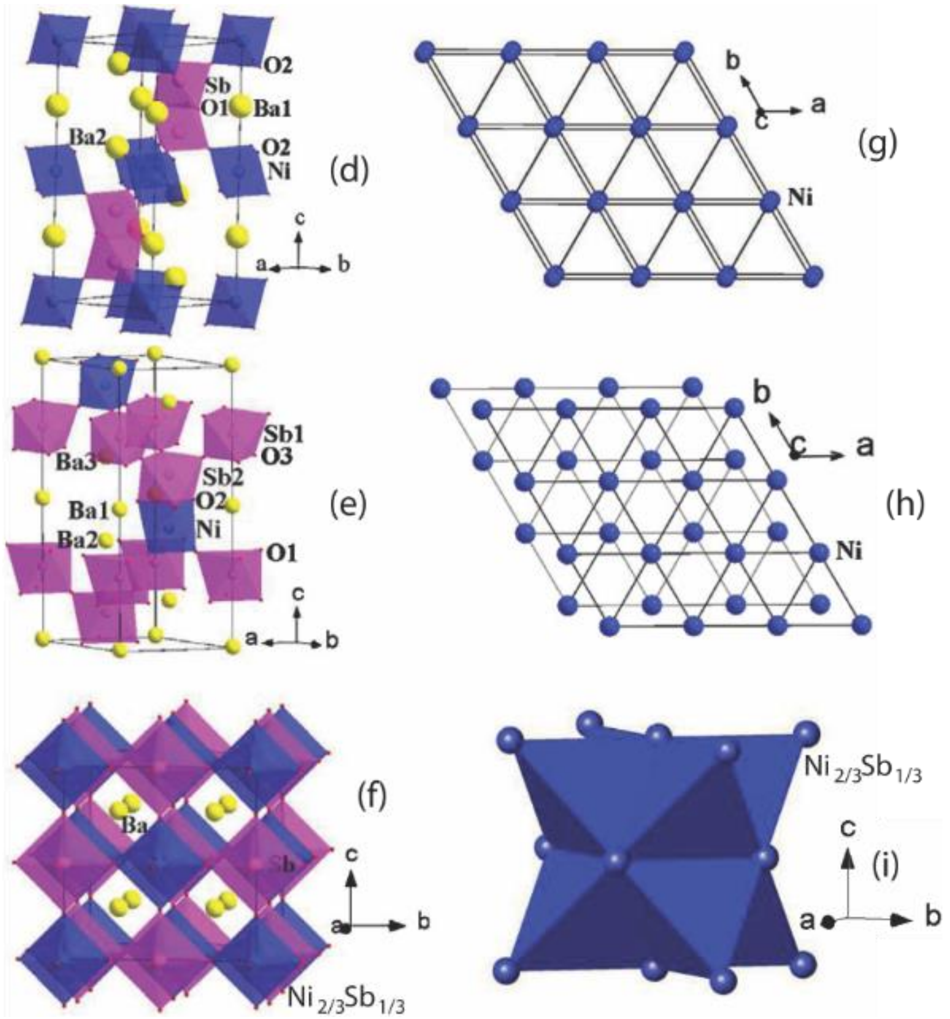
High-Pressure Sequence of $Ba_3NiSb_2O_9$ Structural Phases: New S = 1 Quantum Spin Liquids Based on Ni^{2+}

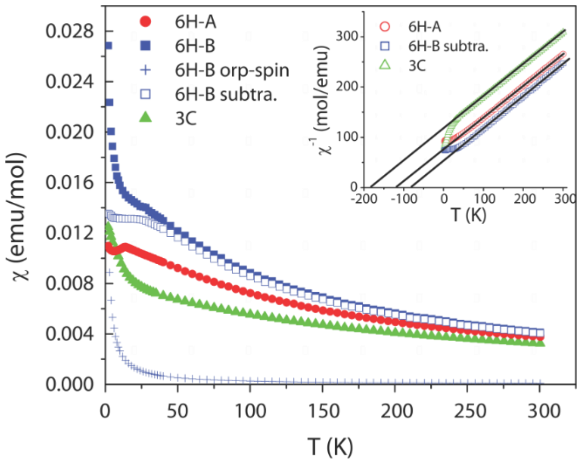
J. G. Cheng,[1] G. Li,[2] L. Balicas,[2] J. S. Zhou,[1] J. B. Goodenough,[1] Cenke Xu,[3] and H. D. Zhou[2,*]

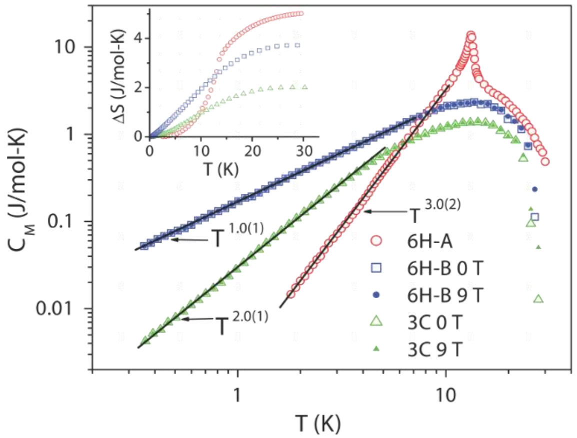
6H-A

6H-B

3C







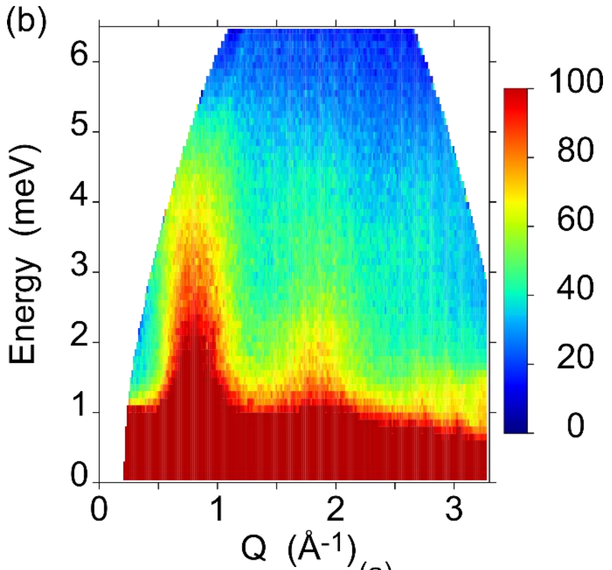
Theories for intriguing 6H-B phase:

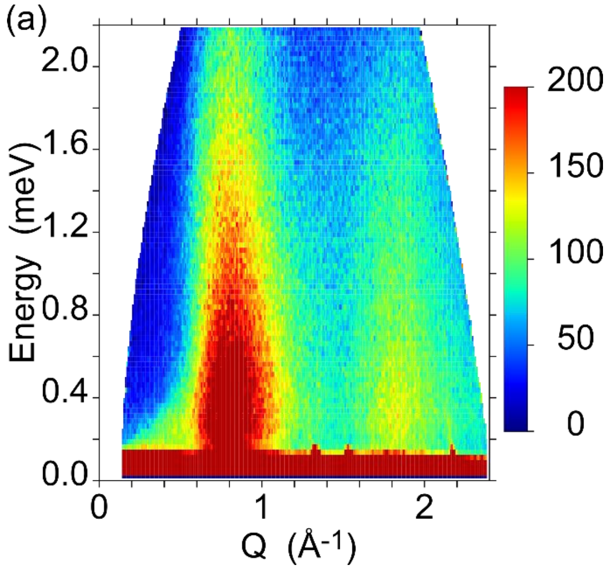
Serbyn et al, PRB 84, 180403 (2011)
SB et al, PRB 86, 224409 (2012)
Xu et al, PRL 108, 087204 (2012)
Chen et al, PRL 109, 016402 (2012)
Hwang et al, PRB 87, 235103 (2013)

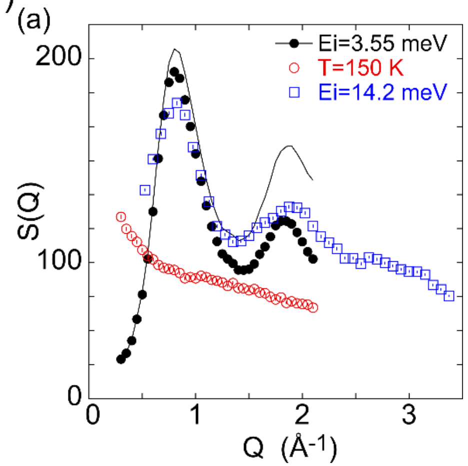
$B\text{-}Ba_3NiSb_2O_9$ (INS)

B. Fåk, **SB**, et al., Phys. Rev. B 95, 060402(R) (2017).

Inelastic neutron scattering on 6H-B phase (powder):





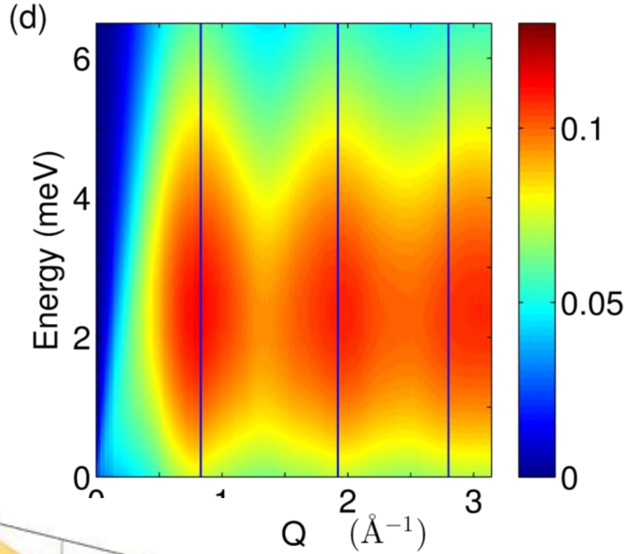


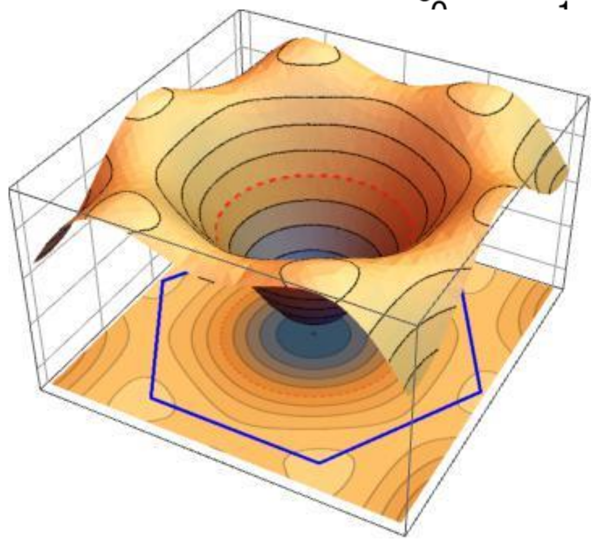
NMR: Quilliam et al, PRB 93, 214432 (2016).

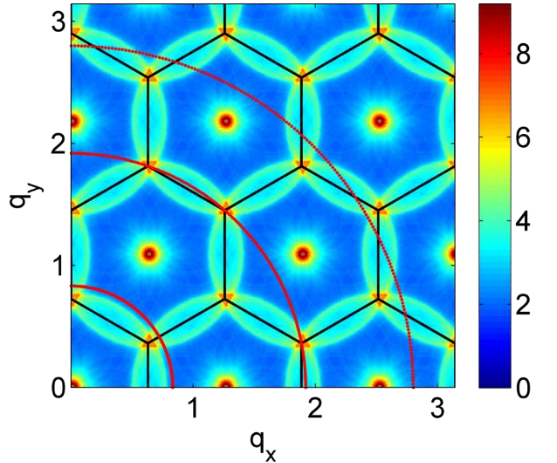
spin fractionalization:

$$\boldsymbol{S} = i\boldsymbol{f}^\dagger \wedge \boldsymbol{f} = i(\varepsilon^{abc} f_b^\dagger f_c),\ a = x, y, z$$

$S(Q,\omega)$ for Fermi sea of spinons at 1/3 filling







Conclusion & outlook

- PSG classification for QSLs
- Exhaustive list of fermonic parton CSLs (kagome, triangular)
- Quasi-1D phase in a S=1/2 kagome system (kapellasite)
- New CSL in kagome Heisenbg. model with DM term (herbertsmithite)
- Evidence for spinon Fermi surface in S=1 triangular QSL (B-BaNiSbO)
- Outlook:
  - Fermionic approach to Dzyaloshinskii-Moriya
  - Ring exchange, honeycomb
  - 3D lattices (hyperkagome, ...)

Thank you!

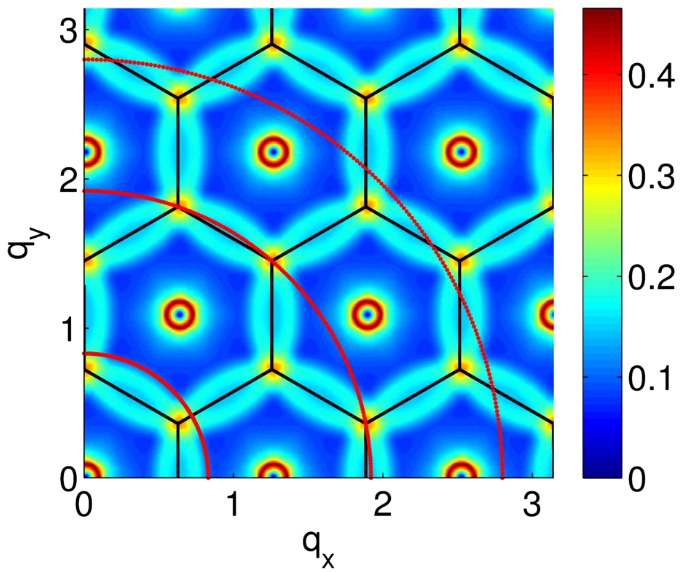
Characterization: Spinon spectrum; Spin structure factor

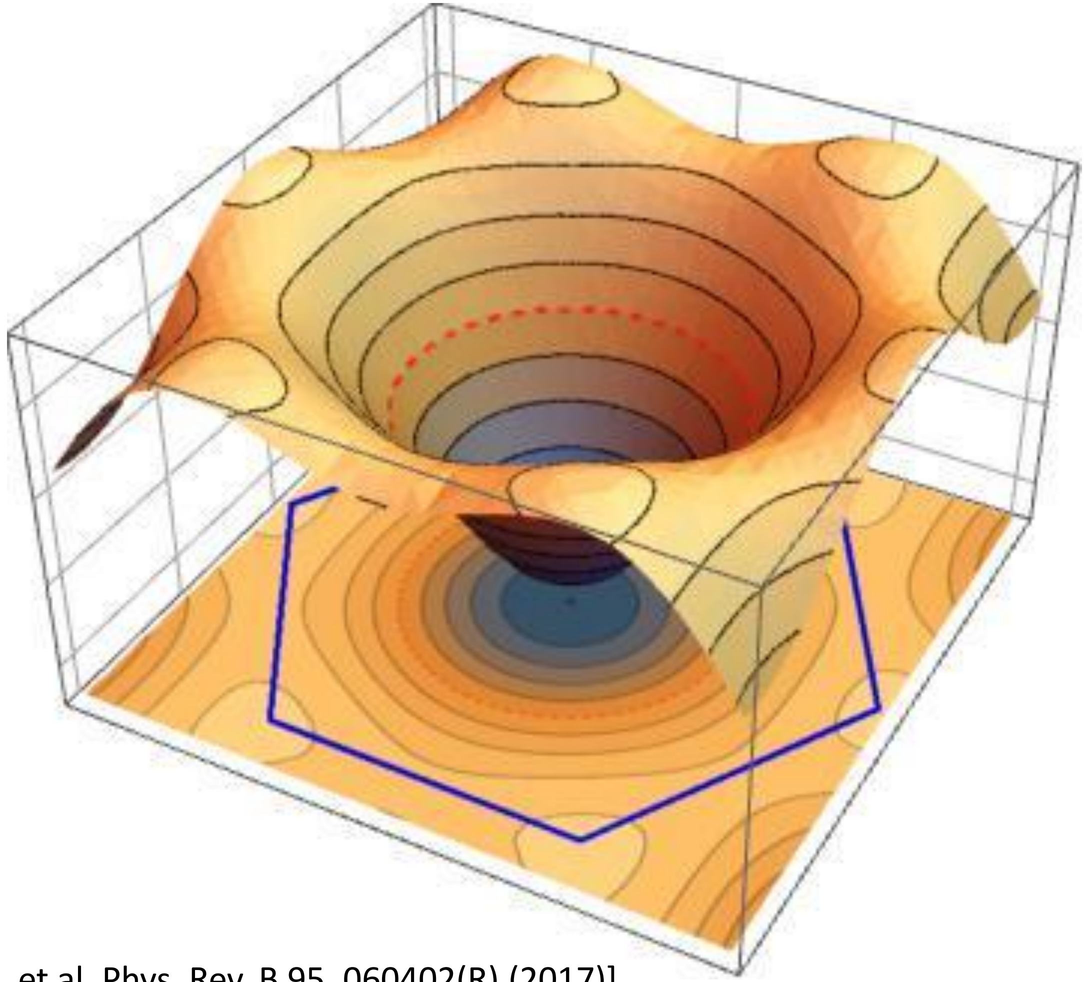
E.g. spinon Fermi surface
(note: This is a Mott insulator!)

$$S \sim \langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle$$

$$S(\boldsymbol{q}) \sim |\boldsymbol{q}|^{\alpha}\ , \ \text{as}\ |\boldsymbol{q}| \to 0$$ ("algebraic SL")

$$S(\boldsymbol{q}, \omega \sim 0) \ \ \text{features at}\ \ \boldsymbol{q} \sim 2\boldsymbol{k}_F$$





[B. Fåk, **SB**, et al, Phys. Rev. B 95, 060402(R) (2017)]